Problems with solutions

1. There are 2 envelopes, each containing an amount of money; the
amount of money is either 5, 10 , 20, 40, 80, 160 euros and everybody
knows this. Furthermore, we are told that an envelope contains exactly
twice as much money as the other. The 2 envelopes are shuffled and we
give one envelope to Ali and one to Baba. After both the envelopes are
opened (but the amounts inside the envelopes are kept private), Ali
and Baba are given the opportunity to switch. If both parties want to
switch, we let them. Will they?

SOLUTION

Suppose that Ali opens her envelope and sees $160. In that case, she
knows that she has the greater amount and hence is unwilling to
participate in a trade. Since Ali won't trade when she has $160 Baba
should refuse to switch envelopes when he has $80 for the only time
Ali might trade occurs when Ali has $40 in which case Baba prefers to
keep his original $80. The only person whi is willing to trade is
someone who finds $5 in the envelope, but of course the other side
doesn't want to trade with him.

2. When Robert Campeau made his first bid for Federeted Stores he used
the strategy of a two-tiered tender offer. Pre-takeover price is 100
euros per share. The first tier of the bid offers a higher price, 105
euros per share to the first shareholders until half of the total
shares are tendered. The next 50% of the shares tendered fall into the
second tier; the price paid for these shares is only 90 euros per
share. For fairness, shares are not placed in the different tiers
based on the order in which they are tendered. Rather, everyone gets a
blended price: all the shares tendered are placed on a prorated basis
into the two tiers. Those who don't tender find all of their shares
end up in the second tier if the bid succeeds. Imagine that another
raider comes along, namely Macy's. Macy's makes a conditional tender
offer: it offers 102 euros per share
provided it gets a majority of the shares. To whom do you tender, and
which (if either) offer do you expect to succeed?

SOLUTION

Tendering to the two-tiered offer is a dominant strategy. To verify
this, we consider all possible cases. There are three possibilities to
check.

1. The two-tiered offer attracts < 50% and fails
2. The two-tiered offer attracts > 50% and succeeds
3. The two-tiered offer attracts = 50%. If you tender the offer will
succeed, and without you it fails.

In the first case if you tender you get $105 which is bigger that
either alternative ($100 or $102).
In the second case if you don't tener you get $90 per share. Tendering
gives you at worst $97.50. So again it is better to tender.
In the third case, while other people are worse off if the offer
succeeds, you are privately better off. The reason is that since there
are exactly 50% tendered you will be getting $105 per share.
Because tendering is a dominant strategy, we expect everyone to
tender. When everyone tenders the average blended price per share may
be below the pre-bid price and even below the expected future price
should the offer fail. Hence the two-tiered bid enables a raider to
pay less than the company is worth.

3. Three antagonists, Larry, Moe and Curly are engaged in a three-way
duel. There are two rounds. In the first round, each player is given
one shot: first Larry, then Moe, and then Curly. After the first
round, any survivors are given a second shot, again beginning with
Larry, then Moe, and then Curly. For each of the duelist, the best
outcome is to be the sole survivor. Next best is to be one of the two
survivors. In third place is the outcome in which no one gets killed.
Dead last is that you get killed. Larry is a poor shot, with only a
30% chance of hitting a person at whom he aims. Moe is a much better
shot, achieving 80% accuracy. Curly is a perfect shot,
he never misses. What is Larry's optimal strategy in the first round?
Who has the gratest chance of survival in this problem?

SOLUTION

Although backward reasoning is the safe way to solve this problem we
can jump ahead a little by using some forward-looking arguments. If
Larry shoots at Moe and hits, then he signs his own death warrant. It
becomes Curly's turn to shoot and he never misses. If Larry hits
Curly, his chance of survival is less than 20%, the chance that Moe
misses. Larry's best strategy is to fire up in the air. In this case
Moe will shoot at Curly and if he misses Curly will shoot and kill
Moe. Then it becomes the second round Larry has at least a 30% chance
of survival. The moral hire is that small fish may do better by
passing on their fist chance to become stars. Your chances of survival
depend on not only your own ability but also whom you threaten.

4. ZECK is a dot game for two players. The object is to force your
opponent to take the last dot. The game starts with dots arranged in
any rectangular shape, for example 7x4:

. . . . . . .
. . . . . . .
. . . . . . .
. . . . . . .

Each turn, a player removes a dot and with it all remaining dots to
the northeast. If the first player chooses the fourth dot in the
second row this leaves his opponent with

. . .
. . .
. . . . . . .
. . . . . . .

Each period, at least one dot must be removed. The person who is foced
to take the last dot loses. For any shaped rectangle with more than
one dot, the first player must have a winning strategy. How to prove
it?

SOLUTION

If the second player has a winning strategy, that means that for any
opening move of the first player, the second has a response that puts
him in a winning position. In particular, this means that the second
player must have a winning response even if the first player just
takes the upper-right-hand dot. But no matter how the second player
responds, the board will be left in a configuration that the first
player could have created in his first move. If this is truly a
winning position, the first player should have and could have opened
the game this way. There is nothing the second player can do to the
first that the first player can't do unto him beforehand.

5. Cell phone companies offer plans with a fixed number of minutes per
month. Minutes you don't use are lost, and if you go over, there is a
steep change. The ad promising 800 minutes for 40euros a month will
almost always cost more than 5c/minute. As a result it becomes
difficult, if not impossible to understand and compare prices. Why
does this practice persist?

SOLUTION
The problem is that the company who plays it straight puts itself at a
disadvantage compared to its rivals. The one honest firm would seem to
be charging the highest price when customers do a comparison on
Expedia or similar websites. We are stuck in a bad equilibrium muck
like the one involving the QWERTY keybord. Customers assume that the
prices will include lots of hidden extras. Imagine that a cell phone
company offered a single flatprice per minute. Does 8c/minute beat $40
for 800 minutes (with a 35c per minute surcharge for going over)?
If society wants to improve matters for customers, one way would be to
legislate a change in the convention:require that hotels, car rental
companies and cell phone providers advertise the all-in price paid by
the average customer.

6. An auctioneer invites bids for a dollar. Bidding proceeds in steps
of 5 cents. The highest bidder gets the dollar, but both the highest
and the second highest bidders pay their bids to the auctioneer. How
would you play this game? Imagine that Eli and John are two bidders.
Each has 2.5 dollars in his wallet and each knows the other's cash
supply. That is the outcome of the auction?

SOLUTION:

This is an example of the slippery slope. The game has one equilibrium
in which the first bid is a dollar and there are no further bids. If
the bidding starts at less than a dollar it will stop only when you
run out of money. If Eli and John both knows they own 2.5 dollars the
first person to bid 1.6 dollars wins, because that establishes a
credible commitment to go up to $2.50 ($1.60 is already lost, but it
is worth his while to spend another 90 cents to capture the dollar).
In order to beat $1.50 it suffices to bid $1.60 and nothing less will
do. Once someone bids 70 cents, it is worthwhile for them to go up to
$1.60 and be guaranteed victory. With this commitment no one with a
bid of 60 cents or less finds it worthwhile to challenge. Although the
numbers will change, the conclusion does not depend on there being
just two bidders. But it is crucial
that everyone know everyone else's budget. When budgets are unknown,
as one would expect, an equilibrium will exist only in mixed
strategies.

7. Imagine that parents want each of their children to visit once and
phone twice a week. To give their children the right incentives they
threaten to disinherit any child who fails to meet this quota. The
estate will be evenly devided among all the children who meet this
quota. The children recognize that their parents are unwilling to
disinherit all of them. As a result, they get together and agree to
cut back the number of visits, potentially down to zero. The parents
call you and ask for some help in revising their will. Where there is
a will, there is a way to make it work. But how? You are not allowed
to disinherit all the children.

SOLUTION:

Any child who fails to meet the quota is disinherited. The problem is
what to do if all of them are below the quota. In that case, give all
of the estate to the child who visits the most. This will make the
children's reduced visiting cartel impossible to maintain. We have put
the children into a multiperson dilemma. The smallest amount of
cheating brings a massive reward. A child who makes just one more
phone call increases his/her inheritance from an equal share to 100
percent. The only escape is to go along with the parents' wishes.

8. A majority of homeowners in the US prefer to live in an unarmed
society. But they are willing to buy a gun if they have reason to fear
that criminals will be armed. Many criminals prefer to carry a gun as
one of the tools of their trade. The table below suggests a possibile
ranking of outcomes

                                          criminals
                                   no guns       guns
                      no guns     1,2           4,1
homeowners
                      guns          2,4           3,3

what is the predicted outcome of the game? Does it change if the
players play in sequence instead of making their moves simultaneously?

SOLUTION

To have a gun is a dominant strategy for criminals. By knowing that,
the homeowners also will prefer to be armed. Thus to be armed is a
Nash equilibrium of the game. If the criminals move first and the
homeowners follow, the subgame perfect Nash equilibrium of the game is
not to carry a gun for both parties. You can easly prove that by
writing down the game tree and solve it by means of backward
induction.

9. A duel. Imagine that you and your rival both write down the time at
which you will shoot. The chance of success at time t is p(t) for you
and q(t) for your rival. If the first shot hits, the game is over. If
it misses, then the other person waits to the end and hits with
certaint. When should you shoot?

SOLUTION

Say you knew your rival would act at t=10. You could either act at
9.99 or wait and let your rival take her  chance. If you shoot at
t=9.99, your chance of winning is just about p(10). If you wait, you
will win if your rival fails. The chance of that is 1-q(10). Hence you
should preempt if p(10)>1-q(10). Of course, your rival is doing the
same calculation. If she thinks you are going to preempt at t=9.99,
she would prefer to move first at t=9.98 if q(9.98)>1-p(9.98).
You can see that the condition that determines the time that neither
side wants to preempt is:
p(t)<=1-q(t) and q(t)<=1-p(t).

These are one and the same condition:
p(t)+q(t)<=1.
Thus both sides are willing to wait until p(t)+q(t)=1 and then they
both shoot.

10. A telecom auction. There arre two bidders, AT&T and MCI, and just
two licences, NY and LA. Both firms are interested in both licences,
but there is only one of each. With help from some game theorists the
FCC ran a simultaneous auction. Both NY and LA were up on the auction
block at the same time. The bidding was divided in into rounds. Each
round, players could raise or stay put. The two firms spent millions
of  dollars preparing for the auction. As part of their preparation,
they figured out both their own value for each of the licences and
what they thought their rival's might be. Here are the evaluations

            NY     LA
AT&T      10      9
MCI          9      8

These valuations are known to both parties. Find the best strategy for
both players and the outcome of the game.

SOLUTION:

AT&T bids 1 for NY and 0 for LA. MCI bids 1 for LA and 0 for NY. AT&T
can win one license at a price of 1 or two licences at a combined
price of 17. The true cost of winning the second licence is 16 far
more that its value. Winning one is the better option. Just because
AT&T can beat MCI in both aucitons doesn't mean that AT&T should. This
is a case of tacit coordination. If you put the two auctions in
sequence tacit coordination doesn't work anymore.