Problems with solutions 

1. There are 2 envelopes, each containing an amount of money; the   
amount of money is either 5, 10 , 20, 40, 80, 160 euros and everybody   
knows this. Furthermore, we are told that an envelope contains exactly   
twice as much money as the other. The 2 envelopes are shuffled and we   
give one envelope to Ali and one to Baba. After both the envelopes are   
opened (but the amounts inside the envelopes are kept private), Ali   
and Baba are given the opportunity to switch. If both parties want to   
switch, we let them. Will they?

SOLUTION

Suppose that Ali opens her envelope and sees $160. In that case, she   
knows that she has the greater amount and hence is unwilling to   
participate in a trade. Since Ali won't trade when she has $160 Baba   
should refuse to switch envelopes when he has $80 for the only time   
Ali might trade occurs when Ali has $40 in which case Baba prefers to   
keep his original $80. The only person whi is willing to trade is   
someone who finds $5 in the envelope, but of course the other side   
doesn't want to trade with him.

2. When Robert Campeau made his first bid for Federeted Stores he used   
the strategy of a two-tiered tender offer. Pre-takeover price is 100   
euros per share. The first tier of the bid offers a higher price, 105   
euros per share to the first shareholders until half of the total   
shares are tendered. The next 50% of the shares tendered fall into the   
second tier; the price paid for these shares is only 90 euros per   
share. For fairness, shares are not placed in the different tiers   
based on the order in which they are tendered. Rather, everyone gets a   
blended price: all the shares tendered are placed on a prorated basis   
into the two tiers. Those who don't tender find all of their shares   
end up in the second tier if the bid succeeds. Imagine that another   
raider comes along, namely Macy's. Macy's makes a conditional tender   
offer: it offers 102 euros per share   
provided it gets a majority of the shares. To whom do you tender, and   
which (if either) offer do you expect to succeed?

SOLUTION

Tendering to the two-tiered offer is a dominant strategy. To verify   
this, we consider all possible cases. There are three possibilities to   
check.

1. The two-tiered offer attracts < 50% and fails   
2. The two-tiered offer attracts > 50% and succeeds   
3. The two-tiered offer attracts = 50%. If you tender the offer will   
succeed, and without you it fails.

In the first case if you tender you get $105 which is bigger that   
either alternative ($100 or $102).   
In the second case if you don't tener you get $90 per share. Tendering   
gives you at worst $97.50. So again it is better to tender.   
In the third case, while other people are worse off if the offer   
succeeds, you are privately better off. The reason is that since there   
are exactly 50% tendered you will be getting $105 per share.   
Because tendering is a dominant strategy, we expect everyone to   
tender. When everyone tenders the average blended price per share may   
be below the pre-bid price and even below the expected future price   
should the offer fail. Hence the two-tiered bid enables a raider to   
pay less than the company is worth.

3. Three antagonists, Larry, Moe and Curly are engaged in a three-way   
duel. There are two rounds. In the first round, each player is given   
one shot: first Larry, then Moe, and then Curly. After the first   
round, any survivors are given a second shot, again beginning with   
Larry, then Moe, and then Curly. For each of the duelist, the best   
outcome is to be the sole survivor. Next best is to be one of the two   
survivors. In third place is the outcome in which no one gets killed.   
Dead last is that you get killed. Larry is a poor shot, with only a   
30% chance of hitting a person at whom he aims. Moe is a much better   
shot, achieving 80% accuracy. Curly is a perfect shot,   
he never misses. What is Larry's optimal strategy in the first round?   
Who has the gratest chance of survival in this problem?

SOLUTION

Although backward reasoning is the safe way to solve this problem we   
can jump ahead a little by using some forward-looking arguments. If   
Larry shoots at Moe and hits, then he signs his own death warrant. It   
becomes Curly's turn to shoot and he never misses. If Larry hits   
Curly, his chance of survival is less than 20%, the chance that Moe   
misses. Larry's best strategy is to fire up in the air. In this case   
Moe will shoot at Curly and if he misses Curly will shoot and kill   
Moe. Then it becomes the second round Larry has at least a 30% chance   
of survival. The moral hire is that small fish may do better by   
passing on their fist chance to become stars. Your chances of survival   
depend on not only your own ability but also whom you threaten.

4. ZECK is a dot game for two players. The object is to force your   
opponent to take the last dot. The game starts with dots arranged in   
any rectangular shape, for example 7x4:

. . . . . . .   
. . . . . . .   
. . . . . . .   
. . . . . . .

Each turn, a player removes a dot and with it all remaining dots to   
the northeast. If the first player chooses the fourth dot in the   
second row this leaves his opponent with

. . .   
. . .   
. . . . . . .   
. . . . . . .

Each period, at least one dot must be removed. The person who is foced   
to take the last dot loses. For any shaped rectangle with more than   
one dot, the first player must have a winning strategy. How to prove   
it?

SOLUTION

If the second player has a winning strategy, that means that for any   
opening move of the first player, the second has a response that puts   
him in a winning position. In particular, this means that the second   
player must have a winning response even if the first player just   
takes the upper-right-hand dot. But no matter how the second player   
responds, the board will be left in a configuration that the first   
player could have created in his first move. If this is truly a   
winning position, the first player should have and could have opened   
the game this way. There is nothing the second player can do to the   
first that the first player can't do unto him beforehand.

5. Cell phone companies offer plans with a fixed number of minutes per   
month. Minutes you don't use are lost, and if you go over, there is a   
steep change. The ad promising 800 minutes for 40euros a month will   
almost always cost more than 5c/minute. As a result it becomes   
difficult, if not impossible to understand and compare prices. Why   
does this practice persist?

SOLUTION   
The problem is that the company who plays it straight puts itself at a   
disadvantage compared to its rivals. The one honest firm would seem to   
be charging the highest price when customers do a comparison on   
Expedia or similar websites. We are stuck in a bad equilibrium muck   
like the one involving the QWERTY keybord. Customers assume that the   
prices will include lots of hidden extras. Imagine that a cell phone   
company offered a single flatprice per minute. Does 8c/minute beat $40   
for 800 minutes (with a 35c per minute surcharge for going over)?   
If society wants to improve matters for customers, one way would be to   
legislate a change in the convention:require that hotels, car rental   
companies and cell phone providers advertise the all-in price paid by   
the average customer.

6. An auctioneer invites bids for a dollar. Bidding proceeds in steps   
of 5 cents. The highest bidder gets the dollar, but both the highest   
and the second highest bidders pay their bids to the auctioneer. How   
would you play this game? Imagine that Eli and John are two bidders.   
Each has 2.5 dollars in his wallet and each knows the other's cash   
supply. That is the outcome of the auction?

SOLUTION:

This is an example of the slippery slope. The game has one equilibrium   
in which the first bid is a dollar and there are no further bids. If   
the bidding starts at less than a dollar it will stop only when you   
run out of money. If Eli and John both knows they own 2.5 dollars the   
first person to bid 1.6 dollars wins, because that establishes a   
credible commitment to go up to $2.50 ($1.60 is already lost, but it   
is worth his while to spend another 90 cents to capture the dollar).   
In order to beat $1.50 it suffices to bid $1.60 and nothing less will   
do. Once someone bids 70 cents, it is worthwhile for them to go up to   
$1.60 and be guaranteed victory. With this commitment no one with a   
bid of 60 cents or less finds it worthwhile to challenge. Although the   
numbers will change, the conclusion does not depend on there being   
just two bidders. But it is crucial   
that everyone know everyone else's budget. When budgets are unknown,   
as one would expect, an equilibrium will exist only in mixed   
strategies.

7. Imagine that parents want each of their children to visit once and   
phone twice a week. To give their children the right incentives they   
threaten to disinherit any child who fails to meet this quota. The   
estate will be evenly devided among all the children who meet this   
quota. The children recognize that their parents are unwilling to   
disinherit all of them. As a result, they get together and agree to   
cut back the number of visits, potentially down to zero. The parents   
call you and ask for some help in revising their will. Where there is   
a will, there is a way to make it work. But how? You are not allowed   
to disinherit all the children.

SOLUTION:

Any child who fails to meet the quota is disinherited. The problem is   
what to do if all of them are below the quota. In that case, give all   
of the estate to the child who visits the most. This will make the   
children's reduced visiting cartel impossible to maintain. We have put   
the children into a multiperson dilemma. The smallest amount of   
cheating brings a massive reward. A child who makes just one more   
phone call increases his/her inheritance from an equal share to 100   
percent. The only escape is to go along with the parents' wishes.

8. A majority of homeowners in the US prefer to live in an unarmed   
society. But they are willing to buy a gun if they have reason to fear   
that criminals will be armed. Many criminals prefer to carry a gun as   
one of the tools of their trade. The table below suggests a possibile   
ranking of outcomes

                                          criminals   
                                   no guns       guns   
                      no guns     1,2           4,1   
homeowners   
                      guns          2,4           3,3

what is the predicted outcome of the game? Does it change if the   
players play in sequence instead of making their moves simultaneously?

SOLUTION

To have a gun is a dominant strategy for criminals. By knowing that,   
the homeowners also will prefer to be armed. Thus to be armed is a   
Nash equilibrium of the game. If the criminals move first and the   
homeowners follow, the subgame perfect Nash equilibrium of the game is   
not to carry a gun for both parties. You can easly prove that by   
writing down the game tree and solve it by means of backward   
induction.

9. A duel. Imagine that you and your rival both write down the time at   
which you will shoot. The chance of success at time t is p(t) for you   
and q(t) for your rival. If the first shot hits, the game is over. If   
it misses, then the other person waits to the end and hits with   
certaint. When should you shoot?

SOLUTION

Say you knew your rival would act at t=10. You could either act at   
9.99 or wait and let your rival take her  chance. If you shoot at   
t=9.99, your chance of winning is just about p(10). If you wait, you   
will win if your rival fails. The chance of that is 1-q(10). Hence you   
should preempt if p(10)>1-q(10). Of course, your rival is doing the   
same calculation. If she thinks you are going to preempt at t=9.99,   
she would prefer to move first at t=9.98 if q(9.98)>1-p(9.98).   
You can see that the condition that determines the time that neither   
side wants to preempt is:   
p(t)<=1-q(t) and q(t)<=1-p(t).

These are one and the same condition:   
p(t)+q(t)<=1.   
Thus both sides are willing to wait until p(t)+q(t)=1 and then they   
both shoot.

10. A telecom auction. There arre two bidders, AT&T and MCI, and just   
two licences, NY and LA. Both firms are interested in both licences,   
but there is only one of each. With help from some game theorists the   
FCC ran a simultaneous auction. Both NY and LA were up on the auction   
block at the same time. The bidding was divided in into rounds. Each   
round, players could raise or stay put. The two firms spent millions   
of  dollars preparing for the auction. As part of their preparation,   
they figured out both their own value for each of the licences and   
what they thought their rival's might be. Here are the evaluations

            NY     LA   
AT&T      10      9   
MCI          9      8

These valuations are known to both parties. Find the best strategy for   
both players and the outcome of the game.

SOLUTION:

AT&T bids 1 for NY and 0 for LA. MCI bids 1 for LA and 0 for NY. AT&T   
can win one license at a price of 1 or two licences at a combined   
price of 17. The true cost of winning the second licence is 16 far   
more that its value. Winning one is the better option. Just because   
AT&T can beat MCI in both aucitons doesn't mean that AT&T should. This   
is a case of tacit coordination. If you put the two auctions in   
sequence tacit coordination doesn't work anymore.