

EXERCISES

1. There are 2 envelopes, each containing an amount of money; the amount of money is either 5, 10, 20, 40, 80, 160 euros and everybody knows this. Furthermore, we are told that an envelope contains exactly twice as much money as the other. The 2 envelopes are shuffled and we give one envelope to Ali and one to Baba. After both the envelopes are opened (but the amounts inside the envelopes are kept private), Ali and Baba are given the opportunity to switch. If both parties want to switch, we let them. Will they?

2. When Robert Campeau made his first bid for Federated Stores he used the strategy of a two-tiered tender offer. Pre-takeover price is 100 euros per share. The first tier of the bid offers a higher price, 105 euros per share to the first shareholders until half of the total shares are tendered. The next 50% of the shares tendered fall into the second tier; the price paid for these shares is only 90 euros per share. For fairness, shares are not placed in the different tiers based on the order in which they are tendered. Rather, everyone gets a blended price: all the shares tendered are placed on a prorated basis into the two tiers. Those who don't tender find all of their shares end up in the second tier if the bid succeeds. Imagine that another raider comes along, namely Macy's. Macy's makes a conditional tender offer: it offers 102 euros per share provided it gets a majority of the shares. To whom do you tender, and which (if either) offer do you expect to succeed?

3. Three antagonists, Larry, Moe and Curly are engaged in a three-way duel. There are two rounds. In the first round, each player is given one shot: first Larry, then Moe, and then Curly. After the first round, any survivors are given a second shot, again beginning with Larry, then Moe, and then Curly. For each of the duelist, the best outcome is to be the sole survivor. Next best is to be one of the two survivors. In third place is the outcome in which no one gets killed. Dead last is that you get killed. Larry is a poor shot, with only a 30% chance of hitting a person at whom he aims. Moe is a much better shot, achieving 80% accuracy. Curly is a perfect shot, he never misses. What is Larry's optimal strategy in the first round? Who has the greatest chance of survival in this problem?

4. ZECK is a dot game for two players. The goal is to force your opponent to take the last dot. The game starts with dots arranged in any rectangular shape, for example 7x4:

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Each turn, a player removes a dot and with it all remaining dots to the northeast. If the first player chooses the fourth dot in the second row this leaves his opponent with

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Each period, at least one dot must be removed. The person who is forced to take the last dot loses. For any shaped rectangle with more than one dot, the first player must have a winning strategy. How to prove it?

5. Cell phone companies offer plans with a fixed number of minutes per month. Minutes you don't use are lost, and if you go over, there is a steep charge. The ad promising 800 minutes for 40 euros a month will almost always cost more than 5c/minute. As a result it becomes difficult, if not impossible to understand and compare prices. Why does this practice persist?

6. An auctioneer invites bids for a dollar. Bidding proceeds in steps of 5 cents. The highest bidder gets the dollar, but both the highest and the second highest bidders pay their bids to the auctioneer. How would you play this game? Imagine that Eli and John are two bidders. Each has 2.5 dollars in his wallet and each knows the other's cash supply. That is the outcome of the auction?

7. Imagine that parents want each of their children to visit once and phone twice a week. To give their children the right incentives they threaten to disinherit any child who fails to meet this quota. The estate will be evenly divided among all the children who meet this quota. The children recognize that their parents are unwilling to disinherit all of them. As a result, they get together and agree to cut back the number of visits, potentially down to zero. The parents call you and ask for some help in revising their will. Where there is a will, there is a way to make it work. But how? You are not allowed to disinherit all the children.

8. A majority of homeowners in the US prefer to live in an unarmed society. But they are willing to buy a gun if they have reason to fear that criminals will be armed. Many criminals prefer to carry a gun as one of the tools of their trade. The table below suggests a possible ranking of outcomes

		Criminals	
		no guns	guns
homeowners	no guns	1,2	4,1
	guns	2,4	3,3

what is the predicted outcome of the game? Does it change if the players play in sequence instead of making their moves simultaneously?

9. A duel. Imagine that you and your rival both write down the time at which you will shoot. The chance of success at time t is $p(t)$ for you and $q(t)$ for your rival. If the first shot hits, the game is over. If it misses, then the other person waits to the end and hits with certainty. When should you shoot?

10. A telecom auction. There are two bidders, AT&T and MCI, and just two licenses, NY and LA. Both firms are interested in both licenses, but there is only one of each. With help from some game theorists the FCC ran a simultaneous auction. Both NY and LA were up on the auction block at the same time. The bidding was divided in into rounds. Each round, players could raise or stay put. The two firms spent millions of dollars preparing for the auction. As part of their preparation, they figured out both their own value for each of the licenses and what they thought their rival's might be. Here are the evaluations

	NY	LA
AT&T	10	9
MCI	9	8

These valuations are known to both parties. Find the best strategy for both players and the outcome of the game.

<http://gregmankiw.blogspot.com/2009/04/exercise-in-game-theory.html>

http://www.cambridge.org/journals/nisan/downloads/Nisan_Non-printable.pdf